In this section we will test the FDM approach for European option pricing and compare it with the previous methods.

Our test results are reported in the following table.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Batch | Type | Exact | N (time step) | J (space step) | result | error |
| 1 | Put | 5.84628 | 1000 | 5K=325 | unstable |  |
| 1 | Put | 5.84628 | 10000 | 5K | 5.84207 | 4.21E-3 |
| 1 | Put | 5.84628 | 100000 | 5K | 5.84205 | 4.23E-3 |
| 1 | Put | 5.84628 | 1000 | 10K=650 | unstable |  |
| 1 | Put | 5.84628 | 10000 | 10K | 5.84525 | 1.03E-3 |
| 1 | Put | 5.84628 | 100000 | 10K | 5.84522 | 1.06E-3 |
| 1 | Put | 5.84628 | 1000 | 20K=1300 | unstable |  |
| 1 | Put | 5.84628 | 10000 | 20K | unstable |  |
| 1 | Put | 5.84628 | 100000 | 20K | 5.84582 | 4.6E-4 |
| 2 | Put | 7.96557 | 1000 | 5K=500 | unstable |  |
| 2 | Put | 7.96557 | 10000 | 5K | 7.96321 | 2.36E-3 |
| 2 | Put | 7.96557 | 100000 | 5K | 7.96312 | 2.45E-3 |
| 2 | Put | 7.96557 | 1000 | 10K=1000 | unstable |  |
| 2 | Put | 7.96557 | 10000 | 10K | unstable |  |
| 2 | Put | 7.96557 | 100000 | 10K | 7.96496 | 6.1E-4 |
| 2 | Put | 7.96557 | 1000 | 20K=2000 | unstable |  |
| 2 | Put | 7.96557 | 10000 | 20K | unstable |  |
| 2 | Put | 7.96557 | 100000 | 20K | 7.96531 | 2.6E-4 |
| 3 | Put | 4.07326 | 1000 | 5K=50 | 4.07113 | 2.13E-3 |
| 3 | Put | 4.07326 | 10000 | 5K | 4.07129 | 1.97E-3 |
| 3 | Put | 4.07326 | 1000 | 10K=100 | Unstable |  |
| 3 | Put | 4.07326 | 10000 | 10K | 4.07273 | 5.3E-4 |
| 3 | Put | 4.07326 | 1000 | 20K=200 | Unstable |  |
| 3 | Put | 4.07326 | 10000 | 20K | 4.07302 | 2.4E-4 |
| 4 | Put | 1.24750 | 1000 | 3K=300 | unstable |  |
| 4 | Put | 1.24750 | 10000 | 3K | unstable |  |
| 4 | Put | 1.24750 | 250000 | 3K | 1.19565 | 5.18E-2 |
| 4 | Put | 1.24750 | 1000 | 5K=500 | unstable |  |
| 4 | Put | 1.24750 | 10000 | 5K | unstable |  |
| 4 | Put | 1.24750 | 500000 | 5K | 1.19586 | 5.17E-2 |

From the results above, we can draw some conclusions about FDM.

First, this method is generally more accurate and time-saving than the MC method, as long as the scheme is stable. FDM performs well except for the batch 4, where the time T is very long.

Second, when the time step can guarantee a stable result, the increase of number of time steps will not increase the accuracy. However, the increase in space step will make the result more accurate.

Third, for different problems, even if the space steps are the same, the time step needed to ensure stability is not the same. (i.e. Batch 2 and 4)